6.2 The Binomial Probability Distribution

1. **Factorial**: The product of the first k positive integers is called the factorial, denoted by $k!$.

$$k! = k \cdot (k-1) \cdot (k-2) \cdots 2 \cdot 1; \qquad 1! = 1; \qquad 0! = 1$$

Note that factorials can be found using calculator. (**MATH PRB** #**4**)

2. **Binomial Coefficients**: (This is also the combination of n objects taken x at a time.) If n is a positive integer and x is a nonnegative integer less than or equal to n, then the binomial coefficient, denoted by ${}_nC_x$ or $\binom{n}{x}$, is defined by

$$ {}_nC_x = \frac{n!}{x!(n-x)!} \, . $$

Binomial coefficients can be found using a calculator. (On the home screen, type the number n, then go to **MATH PRB** #**3** and then type the number x.)

3. **Criteria for a Binomial Probability Experiment**: An experiment is said to be a **binomial experiment** provided that is satisfies the following.

   a. The experiment is performed a fixed number of times. Each repetition of the experiment is called a **trial**. (These trials are called **Bernoulli trials**.) Denote the number of trials as n.

   b. Each trial has only two mutually exclusive outcomes **success**, denoted by s, and **failure**, denoted by f. (Note that the term "success" does not necessarily imply a good thing.)

   c. The **trials are independent**. That is, the outcome of one trial will not affect the outcome of the other trials.

   d. The probability of success remains the same from trial to trial. It is called the **success probability**, denoted by p. (Thus, the probability of failure is $(1-p)$.)

4. The random variable X is called a **binomial random variable** and denotes the number of successes in n Bernoulli trials. The possible values of this random variable are $0 \le x \le n$.

5. Note that since each trial is independent, the **probability of each possible outcome** of a binomial experiment (a combination of n s's and f's) can be found by multiplying the corresponding number of p's and $(1-p)$'s.

   A **tree diagram** can help keep track of these outcomes and their probabilities.

6. **The Binomial Probability Formula**:

a. In n Bernoulli trials, the number of outcomes that contain exactly x successes equals the binomial coefficient ${}_nC_x$.

b. Let X be the random variable that denotes the total number of successes in n Bernoulli trials.

c. Because the trials are independent, the probability of each case with x successes (and so $n - x$ failures) is found by multiplying the probabilities. Thus, each case with x successes will have a probability of $p^x(1-p)^{n-x}$ and there are ${}_nC_x$ such cases.

d. Since each of the cases with exactly x successes are mutually exclusive, the probability of $X = x$ is found by adding these probabilities.

e. **Binomial Probability Formula**: The probability distribution for the random variable X which denotes the number success in n independent trials of a binomial experiment with success probability p is given by

$$P(X = x) = {}_nC_x\, p^x(1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n.$$

The random variable X is called a **binomial random variable** and is said to have the **binomial distribution** with parameters n and p.

7. **To find a Binomial Probability Formula** for n Bernoulli trials:
Check that requirements for a binomial experiment are satisfied.
Identify a success.
Determine p, the probability of success.
Determine n, the number of trials.
Apply the formula: $P(X = x) = {}_nC_x\, p^x(1-p)^{n-x}$.

8. **Key phrases to consider**: (See page 302.)

"at least" or "no less than" or "greater than or equal to"	$x \geq \#$
"more than" or "greater than""	$x > \#$
"fewer than" or "less than"	$x < \#$
"no more than" or "at most" or "less than or equal to"	$x \leq \#$
"exactly"	$x = \#$

9. A **calculator** can be used to find a binomial distribution.

a. To find $P(X = x)$ for the random variable X which denotes the number success in n independent trials of a binomial experiment with success probability p follow the steps:

2nd VAR (**DISTR**) then arrow down to #**0 binompdf(** key in the number of trials, *n*, a comma, the probability of success, *p*, a comma, the number of successes *x*, a parenthesis).

$P(X = x)$: #**0 binompdf(*n*, *p*, *x*)**

b. To find the entire probability distribution, follow the steps above but do not enter the number *x*.

c. To find $P(X \leq x)$ for the random variable X which denotes the number success in n Bernoulli trials with success probability p follow the steps:

2nd VAR (**DISTR**) then arrow down to #**A binomcdf(**
key in the number of trials, *n*, a comma, the probability of success, *p*, a comma, the number x, a parenthesis).

$P(X \leq x)$: #**A binomcdf(*n*, *p*, *x*)**

10. Table II in Appendix A gives a table of binomial probabilities and Table III of Appendix A gives a table of cumulative binomial probabilities.

11. The **mean** and **standard deviation** of a binomial random variable (with success probability p and n trials) are $\mu = np$ and $\sigma = \sqrt{np(1-p)}$.
(The formulas found in section 6.1 will also work but these formulas are easier to apply.)

12. **A Binomial Probability Histogram**: As with any probability histogram, the outcomes, *x*, of the random variable, *X*, are placed on the horizontal axis and their associated probabilities, $P(X = x)$ are placed on the vertical axis.

13. Note that, if n is small, a binomial distribution is right skewed if $p < 0.5$, symmetric and approximately bell shaped if $p = 0.5$, and left skewed if $p > 0.5$.

But as the number of trials, n, increases, the probability distribution of the random variable X becomes approximately bell shaped.

As a rule of thumb, **if $np(1 - p) \geq 10$ then the probability distribution is approximately bell shaped**. (So the Empirical Rule can be used if if $np(1 - p) \geq 10$.)

See pages 306 to 308.