SIGNIFICANT FIGURES

For measured numbers, significant figures relate the certainty of the measurement. As the number of significant figures increases, the more certain the measurement. The means for obtaining the measurement also becomes more sophisticated as the number of significant figures increase.<BR<
Scientific notation is the most reliable way of expressing a number to a given number of significant figures. In scientific notation, the power of ten is insignificant. For instance, if one wishes to express the number 2000 to varying degrees of certainty:

2000 $\longrightarrow$ 2×10^3 is expressed to one significant figure
2000 $\longrightarrow$ 2.0×10^3 is expressed to two significant figures
2000 $\longrightarrow$ 2.00×10^3 is expressed to three significant figures
2000 $\longrightarrow$ 2.000×10^3 is expressed to four significant figures

What do these numbers imply as to the certainty? Let's see what the number can be distinguished from:

The number 2000 to one significant figure lies between:

$$1 \times 10^3 = 1000$$
$$2 \times 10^3 = 2000$$
$$3 \times 10^3 = 3000$$

It is a number that lies between 1000 and 3000 -- not very certain, is it.
The number 2000 to two significant figures lies between:

$$1.9 \times 10^3 = 1900$$
$$2.0 \times 10^3 = 2000$$
$$2.1 \times 10^3 = 2100$$

It is a number that lies between 1900 and 2100 -- more certain than before.
The number 2000 to three significant figures lies between:

$$1.99 \times 10^3 = 1990$$
$$2.00 \times 10^3 = 2000$$
$$2.01 \times 10^3 = 2010$$

It is a number that lies between 1990 and 2010 -- more certain, still.
The number 2000 to four significant figures lies between:

$$1.999 \times 10^3$$
$$2.000 \times 10^3$$
$$2.001 \times 10^3$$

It is a number that lies between 1999 and 2001 -- even more certain.
The more significant figures in a measurement, the more sophisticated the means of measurement. You will see this in the laboratory. To measure out 200 mL of a solution using the marking stamped on the side of the beaker is quick and easy, but it is not as certain as a volumetric flask. When using a volumetric flask, care must be taken to bring the meniscus of the solution to a calibrated etch mark on the flask. Also, beakers are inexpensive pieces of glassware; volumetric flasks, because they are calibrated, are much higher in cost.

Being careless with significant figures may result in dire consequences. The following is a true story told to me by a Baltimore County middle school teacher concerning their mishap resulting from not considering the significance of significant figures:
The science teachers at a Baltimore County middle school wished to acquire a steel cube, one cubic centimeter in size to use as a visual aid to teach the metric system. The machine shop they contacted sent them a work order with instructions to draw the cube and specify its dimensions. On the work order, the science supervisor drew a cube and specified each side to be 1.000 cm. When the machine shop received this job request, they contacted the supervisor to double check that each side was to be one centimeter to four significant figures. The science supervisor, not thinking about the "logistics", verified four significant figures. When the finished cube arrived approximately one month later, it appeared to be a work of art. The sides were mirror smooth and the edges razor sharp. When they looked at the "bottom line", they were shocked to see the cost of the cube to be $500! Thinking an error was made in billing, they contacted the machine shop to ask if the bill was really $5.00, and not $500. At this time, the machine shop verified that the cube was to be made to four significant figure specifications. It was explained to the school, that in order to make a cube of such a high degree of certainty, many man hours were used. The cube had to be ground down, and measured with calipers to within a certain tolerance. This process was repeated until three sides of the cube were successfully completed. The number of man hours to prepare the cube cost $500. The science budget for the school was wiped out for the entire year. This school now has a steel cube worth its weight in gold, because it is a very certain cubic centimeter in size.
When handling significant figures in calculations, two rules are applied:
Multiplication and division -- round the final result to the least number of significant figures of any one term, for example:

$$\frac{(15.03)(4.87)}{1.987} = 36.8$$

The answer, 36.8, is rounded to three significant figures, because least number of significant figures was found in the term, 4.87. The other terms, 15.03 and 1.987, each had 4 significant figures.

Addition and subtraction -- round the final result to the least number of decimal places, regardless of the significant figures of any one term, for example:

$$\begin{array}{r} 1.003 \\ 13.45 \\ +\ 0.0057 \\ \hline 14.4587 \end{array} \quad \text{rounds off to} \quad 14.46$$

The answer, 14.4587, was rounded to two decimal places, since the least number of decimal places found in the given terms was 2 (in the term, 13.45).

Suppose more than one mathematical operation is involved in the calculation? Such a calculation may be "deceptive" as to how many significant figures are actually involved. For instance:

$$\frac{(8.34 - 7.84)}{(15.05)(2.01)} = ?$$

The subtraction in the numerator must be performed first to establish the number of significant figures in the numerator. The subtraction results in:

$$\frac{0.50}{(15.05)(2.01)} = 0.017$$

Since the subtraction in the numerator resulted in a number to two significant figures (rounding to two decimal places), and the least number of significant figures in the resulting expression involving multiplication and division is now two significant figures, the final result must be rounded to two significant figures.

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SIGNIFICANT FIGURES

What are significant figures?

All measurements have some degree of uncertainty; how great the uncertainty is depends on both the accuracy of the measuring device and the skill of its operator. On a triple-beam platform balance, the mass of a sample substance can be measured to the nearest 0.1 g; mass differences less than this cannot be detected on this balance. We might therefore indicate the mass of a dime measured on this balance as 2.2 ± 0.1 g; the ±0.1 (read plus or minus 0.1) is a measure of the accuracy of the measurement. It is important to have some indication of how accurately any measurement is made; the ± notation is one way to accomplish this. It is common to drop the ± notation with the understanding that there is uncertainty of at least one unit in the last digit of the measured quantity; that is, measured quantities are reported in such a way that only the last digit is uncertain. All of the digits, including the uncertain one, are called significant digits or, more commonly, significant figures. The number 2.2 has two significant figures, while the number 2.2405 has five significant figures.

How can we determine how many significant numbers a measurement has?

The following rules apply to determining the number of significant figures in a measured quantity:

1. All nonzero digits are significant--457 cm (three significant figures); 0.25 g (two significant figures).

2. Zeros between nonzero digits are significant--1005 kg (four significant figures); 1.03 cm (three significant figures).

3. Zeros to the left of the first nonzero digits in a number are not significant; they merely indicate the position of the decimal point--0.02 g (one significant figure); 0.0026 cm (two significant figures).

4. When a number ends in zeros that are to the right of the decimal point, they are significant--0.0200 g (three significant figures); 3.0 cm (two significant figures).

5. When a number ends in zeros that are not to the right of a decimal point, the zeros are not necessarily significant--130 cm (two or three significant figures); 10,300 g (three, four, or five significant figures). The way to remove this ambiguity is described below.

Use of standard exponential notation avoids the potential ambiguity of whether the zeros at the end of a number are significant (rule 5). For example, a mass of 10,300 g can be written in exponential notation showing three, four, or five significant figures:

1.03×10^4 g (three significant figures)
1.030×10^4 g (four significant figures)
1.0300×10^4 g (five significant figures)

In these numbers all the zeros to the right of the decimal point are significant (rules 2 and 4).

How do we work with significant figures?

In carrying measured quantities through calculations the rule used is that the accuracy of the result is limited by the least accurate measurement.

Addition and Subtraction

When adding or subtracting, the number of digits to the right of the decimal point in the answer is determined by the measurement which has the least number of digits to the right of the decimal point.

e.g. adding:

26.46 ← this has the least digits to the right of the decimal point (2)
+ 4.123
30.583 rounds off to* → 30.58

2 digits to the right
of the decimal point

e.g. subtracting:

26.46
- 4.123
22.337 rounds off to → 22.34

The above rule is based on the fact that the last digit retained in the sum or difference is determined by the first doubtful figure (which is underlined in the following example).

37.24
+10.3
_____
47.54 rounds off to → 47.5

We may report the first doubtful figure but no more.

Multiplication and Division

In multiplying or dividing, the number of significant figures in the answer--regardless of the position of the decimal point equals that of the quantity which has the smaller number of significant figures.

e.g. multiplying:

  2.61
x 1.2   this has the smaller number of significant figures (2)
3.132   rounds off to → 3.1   has 2 significant figures
e.g. dividing: $2.61 \div 1.2 = 2.175$ rounds off to → 2.2

*When rounding off do the following: When the number to be dropped is less than 5, it is just dropped (e.g., 6.34 rounds off to 6.3). When it is more than 5, the preceding number is increased by 1 (e.g., 5.27 rounds off to 5.3). When the number to be dropped is five, the preceding number is not changed when it (the preceding number) is even (i.e., 4.45 rounds off to 4.4). When the preceding number is odd, it is increased by 1 (i.e., 4.35 rounds off to 4.4). In fairness, we must note that the even-odd rules for rounding terminal 5's are sometimes ignored; instead, the preceding number is increased by 1 when a 5 is dropped.

Again this rule has been based on the fact that we may report only one doubtful figure. For example, if we underline each uncertain figure as well as each figure obtained from an uncertain figure the step-by-step multiplication gives

$$
\begin{array}{r}
12.3\underline{4} \\
\times \quad 1.2\underline{3} \\
\hline
\underline{37}\ \underline{02} \\
246\ \underline{8} \\
123\underline{4} \\
\hline
15.\underline{1782} = 15.\underline{2}
\end{array}
$$

← (for $\times$ 1.23) this has the lowest number of significant figures (3)

← (for 15.2) The answer must have 3 significant figures

This handout has used material from the following sources:
Chemistry The Central Science by Brown and LeMay, Prentice Hall, Inc., 1977.
Determining Significant Figures, Series 800. Basic Math for Science Students and Technicians, Prentice-Hall Media. 1972.

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Beaker

Math Skills Review

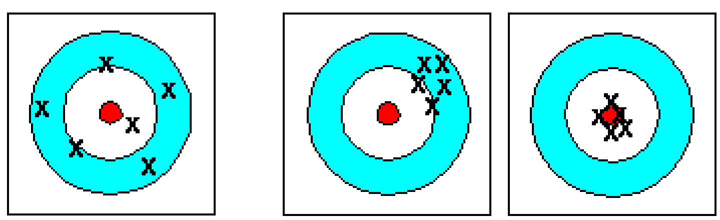
Significant Figures

There are two kinds of numbers in the world**:**

- **exact:**
  - example: There are exactly 12 eggs in a dozen.
  - example: Most people have exactly 10 fingers and 10 toes.
- **inexact numbers:**
  - example: any measurement.
    If I quickly measure the width of a piece of notebook paper, I might get 220 mm (2 significant figures). If I am more precise, I might get 216 mm (3 significant figures). An even more precise measurement would be 215.6 mm (4 significant figures).

PRECISION VERSUS ACCURACY

Accuracy refers to how closely a measured value agrees with the correct value.
Precision refers to how closely individual measurements agree with each other.



accurate (the average is accurate) not precise	precise not accurate	accurate and precise

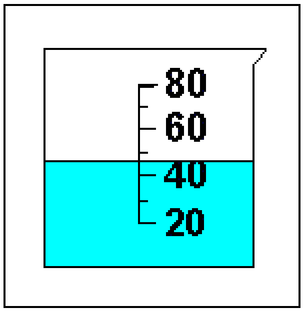
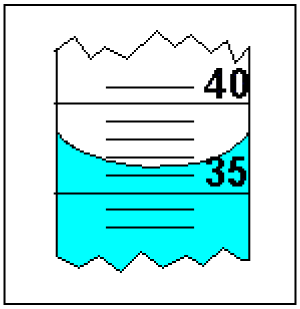
In any measurement, the number of significant figures is critical. The number of significant figures is the number of digits believed to be correct by the person doing the measuring. It includes one **estimated** digit.

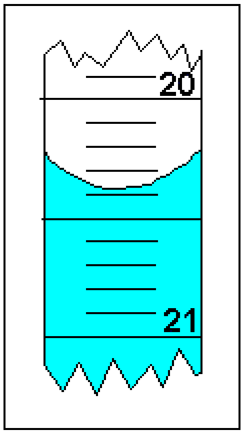
Let's look at an example where significant figures is important**:** measuring volume in the laboratory. This can be done in many ways**:** using

- a beaker with volumes marked on the side,
- a graduated cylinder, or
- a buret.

Which glassware would give you the most precise volume measurement? Let's figure out the volume for each one and its associated error. This will give us the number of figures that are significant. Recall: the number of significant figures includes one estimated digit.

A rule of thumb: read the volume to 1/10 or 0.1 of the smallest division. (This rule applies to any measurement.) This means that the error in reading (called the reading error) is ±1/10 or 0.1 of the smallest division on the glassware. If you are less sure of yourself, you can read to 1/5 or 0.2 of the smallest division.

Beaker		The smallest division is 10 mL, so we can read the volume to ±1/10 of 10 mL or ±1 mL. The volume we read from the beaker has a reading error of ±1 mL. The volume in this beaker is 47 ±1 mL. You might have read 46 mL; your friend might read the volume as 48 mL. All the answers are correct within the reading error of ± 1 mL. So, How many significant figures does our volume of 47 ±1 mL have? Answer - 2! The "4" we know for sure plus the "7" we had to estimate.
Graduated Cylinder	 Look in the textbook for a picture of a graduated cylinder.	First, note that the surface of the liquid is curved. This is called the meniscus. This phenomenon is caused by the fact that water molecules are more attracted to glass than to each other (adhesive forces are stronger than cohesive forces). When we read the volume, we read it at the BOTTOM of the meniscus. The smallest division of this graduated cylinder is 1 mL. Therefore, our reading error will be

		±0.1 mL or 1/10 of the smallest division. An appropriate reading of the volume is 36.5 ±0.1 mL. An equally precise value would be 36.6 mL or 36.4 mL. How many significant figures does our answer have? 3! The "3" and the "6" we know for sure and the "5" we had to estimate a little.
Buret	 Look in the textbook for a picture of a buret. Note that the numbers get bigger as you go down the buret. This is different from the beaker or the graduated cylinder. This is because the liquid leaves the buret at the bottom.	The smallest division in this buret is 0.1 mL. Therefore, our reading error is ±0.01 mL. A good volume reading is 20.38 ±0.01 mL. An equally precise answer would be 20.39 mL or 20.37 mL. How many significant figures does our answer have? 4! The "2", "0", and "3" we definitely know and the "8" we had to estimate.

Conclusion: The number of significant figures is directly linked to a measurement. If a person needed only a rough estimate of volume, the beaker volume is satisfactory (2 significant figures), otherwise one should use the graduated cylinder (3 significant figures) or better yet, the buret (4 significant figures).

Rules for Significant Figures:

1. Leading zeros are never significant.
   Imbedded zeros are always significant.
   Trailing zeros are significant only if the decimal point is specified.
   Hint: Change the number to scientific notation. It is easier to see.
2. **Addition or Subtraction:**
   The last digit retained is set by the first doubtful digit.
3. **Multiplication or Division:**

The answer contains no more significant figures than the **least** accurately known number.

EXAMPLES:

Example	Number of Significant Figures	Scientific Notation	
0.00682	3	6.82×10^{-3}	Leading zeros are not significant.
1.072	4	$1.072\ (\times 10^{0})$	Imbedded zeros are always significant.
300	1	3×10^{2}	Trailing zeros are significant only if the decimal point is specified.
300**.**	3	3.00×10^{2}	
300**.**0	4	3.000×10^{2}	

EXAMPLES

Addition	4.7832 1.234 + 2.02 8.0372 ⇓ rounding 8.04	Even though your calculator gives you the answer 8.0372, you must round off to 8.04. Your answer must only contain 1 doubtful number. Note that the doubtful digits are underlined.
Subtraction	1.0236 - 0.97268 0.05092 ⇓ rounding 0.0509	Subtraction is interesting when concerned with significant figures. Even though both numbers involved in the subtraction have 5 significant figures, the answer only has 3 significant figures when rounded correctly. Remember, the answer must only have 1 doubtful digit.
Multiplication	2.8723 x 1.6 4.59568 ⇓ rounding 4.6	The answer must be rounded off to 2 significant figures, since 1.6 only has 2 significant figures.
Division	45.2 ÷ 6.3578 7.1093775 ⇓ rounding 7.11	The answer must be rounded off to 3 significant figures, since 45.2 has only 3 significant figures.

Notes on Rounding

- When rounding off numbers to a certain number of significant figures, do so to the nearest value.
  - example: Round to 3 significant figures: 2.3467×10^4 (Answer: 2.35×10^4)
  - example: Round to 2 significant figures: 1.612×10^3 (Answer: 1.6×10^3)
- What happens if there is a 5? There is an arbitrary rule:
  - If the number before the 5 is odd, round up.
  - If the number before the 5 is even, let it be.
    The justification for this is that in the course of a series of many calculations, any rounding errors will be averaged out.
  - example: Round to 2 significant figures: 2.35×10^2 (Answer: 2.4×10^2)
  - example: Round to 2 significant figures: 2.45×10^2 (Answer: 2.4×10^2)
  - Of course, if we round to 2 significant figures: 2.451×10^2, the answer is definitely 2.5×10^2 since 2.451×10^2 is closer to 2.5×10^2 than 2.4×10^2.

QUIZ:

Question 1 Give the correct number of significant figures for 4500, 4500**.**, 0.0032, 0.04050

Question 2 Give the answer to the correct number of significant figures:
4503 + 34.90 + 550 = ?

Question 3 Give the answer to the correct number of significant figures:
1.367 - 1.34 = ?

Question 4 Give the answer to the correct number of significant figures:
$(1.3 \times 10^3)(5.724 \times 10^4)$ = ?

Question 5 Give the answer to the correct number of significant figures:
(6305)/(0.010) = ?

Answers: (1) 2, 4, 2, 4 **(2)** 5090 (3 significant figures - round to the tens place - set by 550) **(3)** 0.03 (1 significant figure - round to hundredths place) **(4)** 7.4×10^7 (2 significant figures - set by 1.3×10^3) **(5)** 6.3×10^5 (2 significant figures - set by 0.010)

Pick your next topic:

Algebraic Manipulation · Scientific Notation · Manipulation of Exponents
Dimensional Analysis · Logarithms · The Quadratic Equation