The Graphing Calculator and Quadratic Equations

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Math 0098

There are three primary ways that the graphing calculator can be used in the chapter on quadratic equations and functions. The calculator can be used to approximate irrational solutions to quadratic equations. The calculator can be used to verify solutions to quadratic equations. Lastly, the calculator can be used to graph quadratic functions.

Objective 1: Approximate Irrational Solutions

Irrational solutions may be approximated by entering the appropriate expression on the calculator. Solutions that involve fractions must be entered carefully so that the correct approximation will be found.

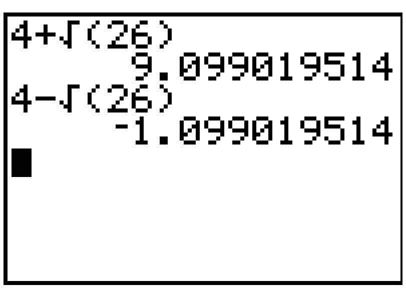
Solve the equations below using any of the methods from chapter 11 and then use your calculator to approximate the solutions to the nearest thousandths.

Example 1:

$$x^2 - 8x - 10 = 0$$
$$x^2 - 8x = 10$$
$$x^2 - 8x + 16 = 10 + 16$$
$$(x-4)^2 = 26$$
$$x - 4 = \pm\sqrt{26}$$
$$x = 4 \pm \sqrt{26}$$
$$x = 4 + \sqrt{26} \text{ or } x = 4 - \sqrt{26}$$

To solve by completing the square:
1) Remove the constant from the left side.
2) Complete the square (half of 8, squared) and add to both sides.
3) Factor the perfect square trinomial on the left.
4) Apply the square root property.
5) Isolate x by adding 4 to both sides.

To approximate the solutions, use the calculator:



The solutions rounded to the nearest thousandth are 9.099 and -1.099.

Example 2:

$$2x^2 - 5x - 6 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-6)}}{2(2)}$$

$$x = \frac{5 \pm \sqrt{25 + 48}}{4}$$

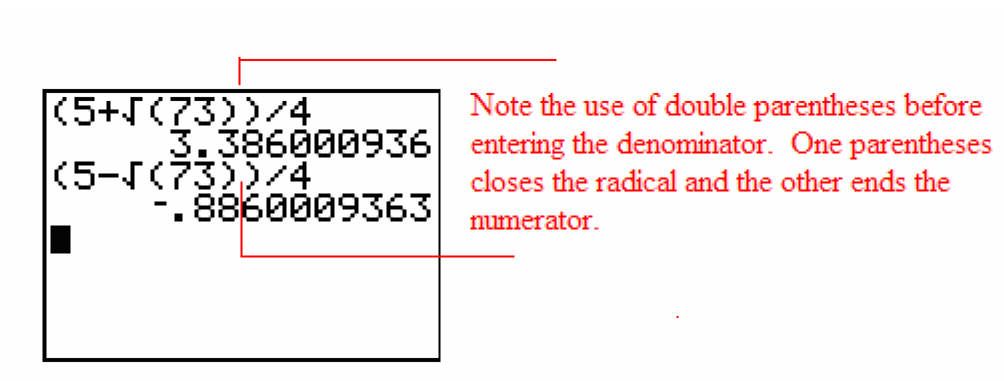
$$x = \frac{5 \pm \sqrt{73}}{4}$$

$$x = \frac{5 + \sqrt{73}}{4} \text{ or } x = \frac{5 - \sqrt{73}}{4}$$

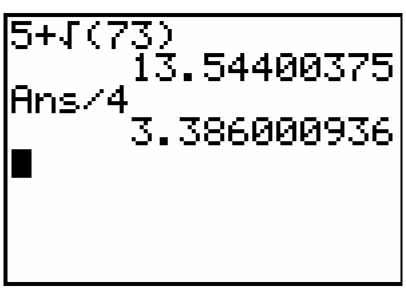
To solve using the Quadratic Formula:

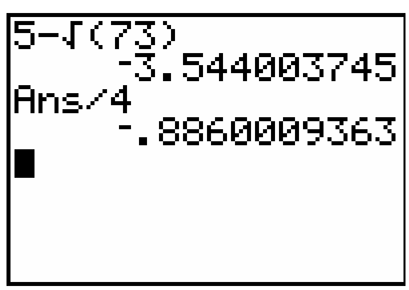
1) Identify the values of a, b, and c.
2) Substitute into the quadratic formula.
3) Simplify the resulting expression.

To use the calculator to approximate the solutions, you must use parentheses to enter the numerator since it is more than a single term.



You could also enter this type of expression in two parts – find the value of the numerator and then divide by the denominator.





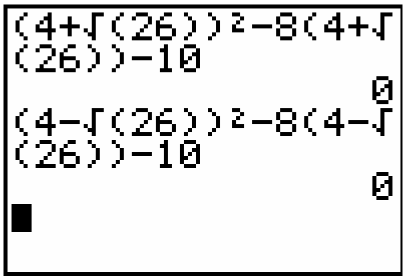
The two approximate solutions, rounded to the nearest thousandth, are 3.386 and -.886.

Objective 2: Use the calculator to verify solutions. (Two methods illustrated.)

Method 1: Recall that a solution to an equation is a number that makes the equation true when this value is substituted for the variable. One way to check your solution is to substitute the value of the variable on your calculator. (I will use the two examples above to illustrate this.)

Examples:

1) Verify that $x = 4 + \sqrt{26}$ or $x = 4 - \sqrt{26}$ are solutions to $x^2 - 8x - 10 = 0$.



When the values were substituted in the left side of the equation, the result is 0 which equals the right side of the equation. Therefore, the solutions we found are correct.

Note: Two parentheses had to be entered after 26. One of these closes the radical; the other closes the value of x.

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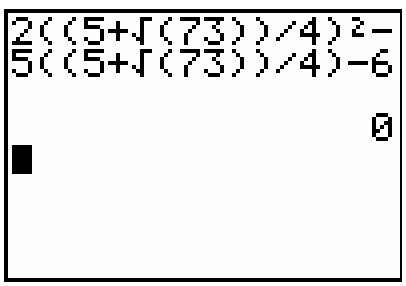
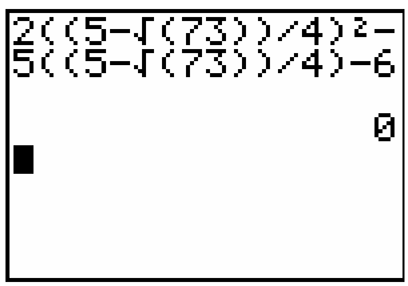
2) Verify that $x = \frac{5+\sqrt{73}}{4}$ or $x = \frac{5-\sqrt{73}}{4}$ are solutions to $2x^2 - 5x - 6 = 0$.





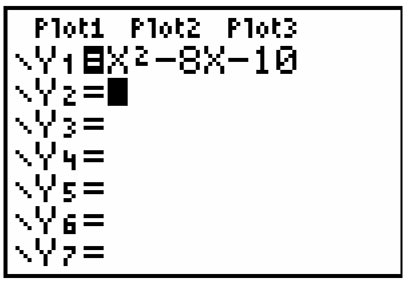
To check these solutions, be very careful about the use of parentheses when inserting the value of the variable. One set of ( ) are used around the entire expression. One set is used around the numerator and one set is used around the radical.

Method 2: When an equation is of the form f(x) = 0, the solutions to the equation actually represent the x-intercepts of the graph of the function f(x). (Recall that to find the x-intercepts of a graph, we let y = 0 and solve for x. This is exactly what we are doing when we solve a quadratic equation.)

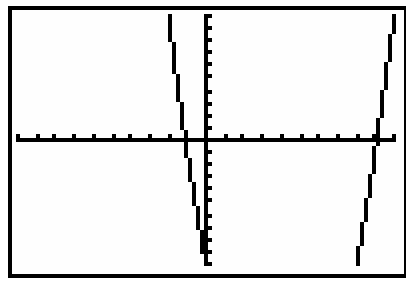
Example 1: Verify that $x = 4 + \sqrt{26}$ or $x = 4 - \sqrt{26}$ are solutions to $x^2 - 8x - 10 = 0$ graphically.

To verify this graphically, let y = $x^2 - 8x - 10$. Graph this equation and then verify that your solutions are the x-intercepts of the graph.

Step 1: Enter the equation in the calculator.

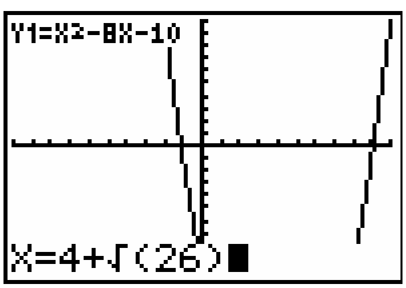


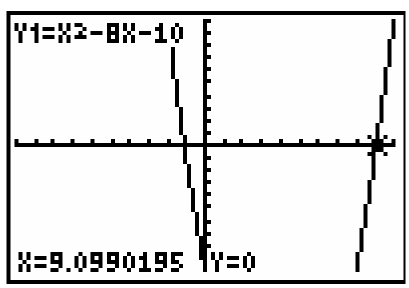
Step 2: Graph the equation.



Step 3: Use the TRACE feature to determine if your solutions are the x-intercepts of the graph.

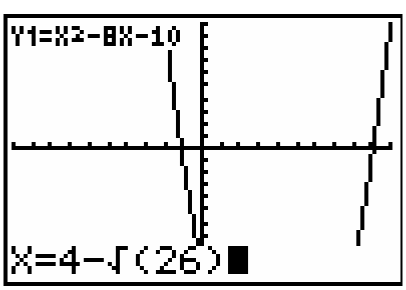
Press TRACE and then enter the value of x and press enter.

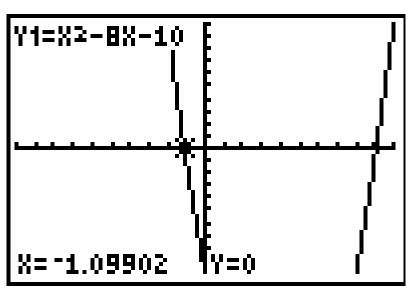




Note that the cursor moved to the x-intercept of the graph for this x-value. This confirms that our solution is correct.

Repeat this for the other solution.

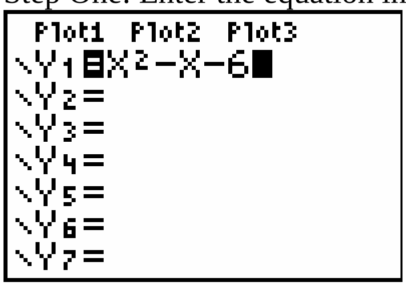




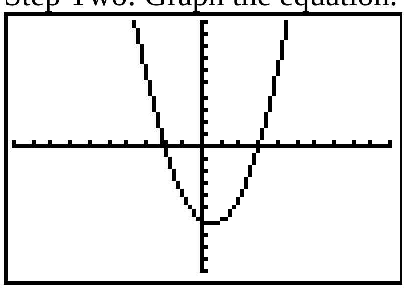
Example 2: Solve the equation $x^2 - x - 6 = 0$ by hand and then confirm your solutions graphically.

Solution:
$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$
$$x - 3 = 0 \text{ or } x + 2 = 0$$
$$x = 3 \qquad \text{or} \qquad x = -2$$

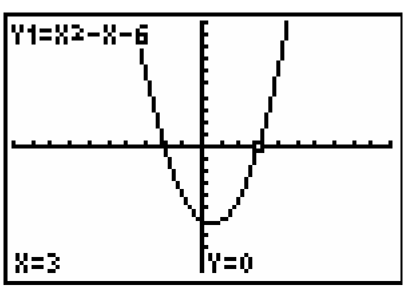
To verify graphically:
Step One: Enter the equation in the calculator.

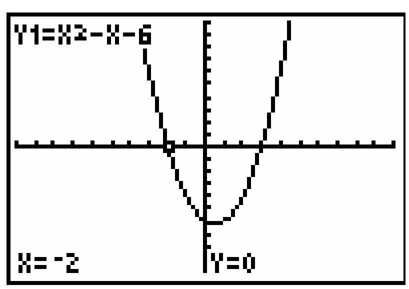


Step Two: Graph the equation.



Step Three: Confirm that the x-intercepts are (3, 0) and (-2, 0). Press TRACE and enter the two x-values.





Since the graph crosses the x-axis at the x-values of 3 and -2, our solutions are correct.

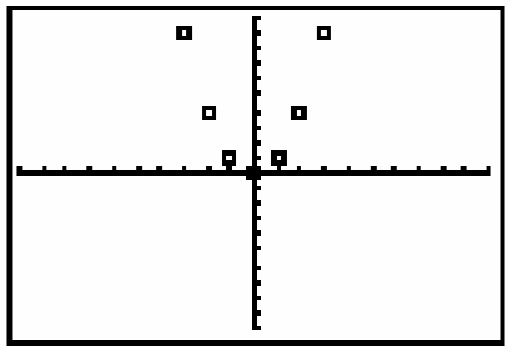
Objective 3: Graph a quadratic function.

The graph of an equation of the form $y = ax^2 + bx + c$ or $y = a(x-h)^2 + k$ is the graph of a quadratic function. The graph of this type of function is a parabola.

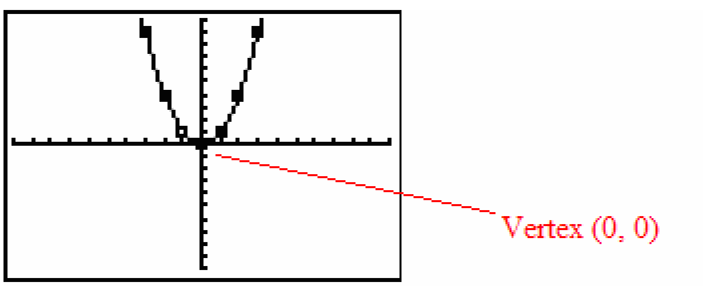
The basic graph of a quadratic equation is the graph of $y = x^2$.

This graph of this function can be found by hand by plotting points. Substitute some values for x in the equation to find their corresponding y values and then plot the resulting points to determine the pattern of the solutions.

X	$y = x^2$	(x, y)
-3	$y = (-3)^2 = 9$	(-3, 9)
-2	$y = (-2)^2 = 4$	(-2, 4)
-1	$y = (-1)^2 = 1$	(-1, 1)
0	$y = (0)^2 = 0$	(0, 0)
1	$y = (1)^2 = 1$	(1, 1)
2	$y = (2)^2 = 4$	(2, 4)
3	$y = (3)^2 = 9$	(3, 9)

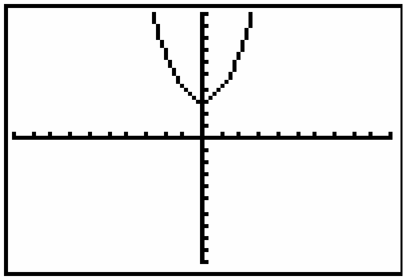
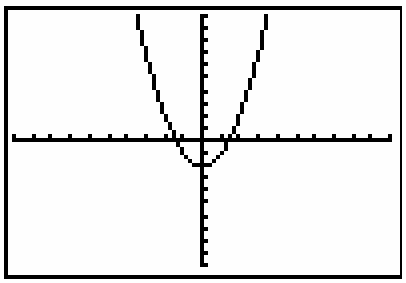


These points are in the shape of a parabola. Connecting the points with a smooth curve we get

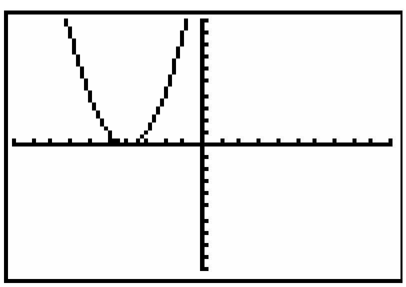
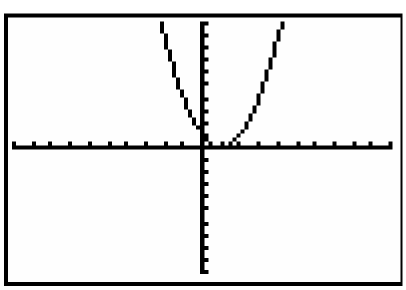


The graph of a quadratic equation will always be a parabola. The vertex is the top-most or bottom-most point on the parabola. Note also that if you fold the parabola vertically at the vertex point, the left is a mirror image of the right.

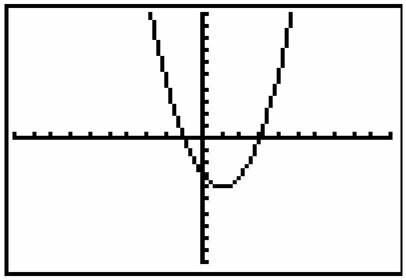
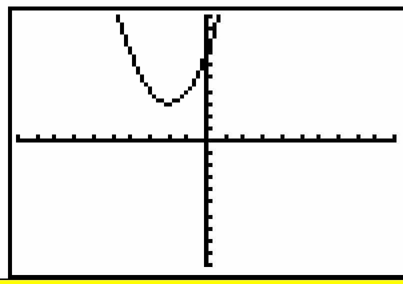
Now let's look at the graph of some other parabolas and see how their graph relates to the graph of $y = x^2$.

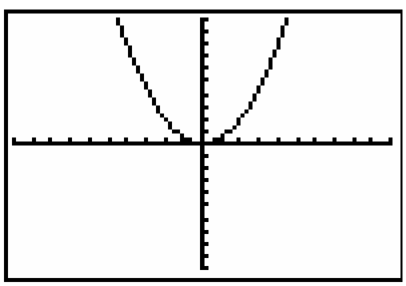
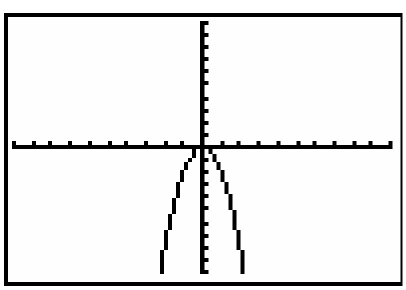
Equation	Graph	Vertex	Relationship to $y = x^2$
$y = x^2 + 3$		(0, 3)	The graph of this equation is the graph of $y = x^2$ shifted up three units.
$y = x^2 - 2$		(0, -2)	The graph of this equation is the graph of $y = x^2$ shifted down two units.

Conclusion: The graph of $y = x^2 + k$ is the graph of $y = x^2$ shifted up k units (if k is positive) or shifted down k units (if k is negative). The vertex of this equation is (0, k).

$y = (x+4)^2$		(-4, 0)	The graph of this equation is the graph of $y = x^2$ shifted four units to the left.
$y = (x-1)^2$		(1, 0)	The graph of this equation is the graph of $y = x^2$ shifted one unit right.

Conclusion: The graph of the equation $y = (x-h)^2$ is the graph of $y = x^2$ shifted left h units (if h is positive) or shifted right h units (if h is negative). The vertex of this equation is (h, 0).

Equation	Graph	Vertex	Relationship to $y = x^2$
$y = (x-1)^2 - 4$		(1, -4)	The graph of this equation is the graph of $y = x^2$ shifted one unit right and four units down.
$y = (x+2)^2 + 3$		(-2, 3)	The graph of this equation is the graph of $y = x^2$ shifted two units left and three units up.
Conclusion: The graph of the equation $y = (x-h)^2 + k$ is the graph of $y = x^2$ shifted left/right h units and up/down k units. The vertex of the graph is (h, k).			

$y = \frac{1}{2}x^2$		(0, 0)	The graph of this equation is the graph of $y = x^2$ compressed by a factor of ½. Basically this means that the graph has been widened. (Original y values are cut in half.)
$y = -2x^2$		(0, 0)	The graph of this equation is the graph of $y = x^2$ reflected over the x-axis and stretched by a factor of 2. In other words, the graph is going down and the y-values have been doubled and negated.
Conclusion: The graph of $y = ax^2$ is the graph of $y = x^2$ either stretched or compressed. If the value of a is positive, then the graph opens up. If the graph of a is negative, the graph opens down. The vertex of this type of equation is (0, 0) since the parabola did not shift up/down or left/right.			

SUMMARY:
To graph the equation $y = a(x-h)^2 + k$,

1) Identify the vertex (h, k).
2) Determine if the parabola opens up or down. If a > 0, open up. If a < 0, opens down.
3) Plot two points in addition to the vertex – one of the left side of the parabola and one on the right side.
4) Connect the three points in the shape of a parabola.

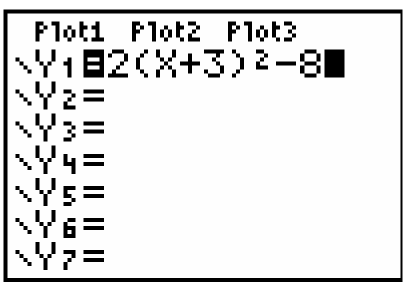
Example: Graph the equation $y = 2(x+3)^2 - 8$.

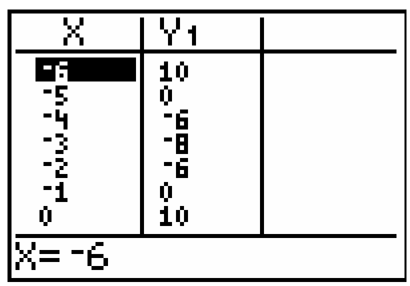
Solution:
1) The vertex of the parabola is (-3, -8).
2) Since a = 2, the parabola opens up.
3) Find two additional points. You can do this by hand or on the calculator.

By hand: Let x = -5 and x = -1 (a value to the left of -3 and to the right of -3)

x = -5: $y = 2(-5+3)^2 - 8 = 2(-2)^2 - 8 = 2(4) - 8 = 8 - 8 = 0$ (-5, 0)
x = -1: $y = 2(-1+3)^2 - 8 = 2(2)^2 - 8 = 2(4) - 8 = 8 - 8 = 0$ (-1, 0)

By the calculator:





4) Plot the three points and draw the parabola.