

Solving Systems of Equations by Substitution

Same goal - find point of intersection of the lines in the system

Possible solutions - one ordered pair, no solution, or infinitely many solutions

Graphing has its limitations so we need an algebraic method to solve a linear system.

Method of Substitution:

Solve one equation in the system for one of the variables.

Substitute this expression into the other equation.

Solve the resulting equation.

Now find the value of the other coordinate of the solution.

Example: Solve $x + 3y = -28$ by substitution.

$$y = -5x$$

$$\begin{aligned}x + 3y &= -28 \\ y &= -5x\end{aligned}$$

① $y = -5x$

② $x + 3y = -28$
 $x + 3(-5x) = -28$

$$\begin{aligned}1x - 15x &= -28 \\ -14x &= \frac{-28}{-14}\end{aligned}$$

③ $x = 2$

③ $y = -5x$
 $y = -5(2)$
 $y = -10$

④ $\{(2, -10)\}$

Example: Solve $3x - y = 5$ by substitution.

$$y = 3x - 5$$

$$\left. \begin{array}{l} 3x - y = 5 \\ y = 3x - 5 \end{array} \right\} \text{same line}$$

① $y = 3x - 5$

② $3x - (3x - 5) = 5$

$$\begin{array}{r} \cancel{3x} - \cancel{3x} + 5 = 5 \\ \underline{5 = 5} \end{array}$$

True

③ infinitely many

$$\{(x, y) \mid 3x - y = 5\}$$

Example: Solve $2x + 8y = 3$ by substitution.

$$x = 8 - 4y$$

$$2x + 8y = 3$$
$$x = 8 - 4y$$

① $x = 8 - 4y$

② $2(8 - 4y) + 8y = 3$

$$16 - 8y + 8y = 3$$

$$16 = 3 \text{ False}$$

③ parallel lines \rightarrow no sol. \emptyset

Example: Solve $x/5 + y = 6/5$
 $x/10 + y/3 = 5/6$

by substitution.

$$5 \left(\frac{x}{5} + y = \frac{6}{5} \right) \rightarrow \underline{x} + 5y = 6$$

$$30 \left(\frac{x}{10} + \frac{y}{3} = \frac{5}{6} \right) \rightarrow 3x + 10y = 25$$

$$\left\{ \left(13, -\frac{7}{5} \right) \right\}$$

$$\textcircled{1} \quad x + 5y = 6$$

$$x = 6 - 5y$$

$$-5y = 7$$

$$y = -\frac{7}{5}$$

$$\textcircled{2} \quad 3(6 - 5y) + 10y = 25$$

$$18 - 15y + 10y = 25$$

$$-18 - 5y = 25 - 18$$

$$\textcircled{3} \quad x + 5y = 6$$

$$x + 5\left(-\frac{7}{5}\right) = 6$$

$$x - 7 = 6$$

$$x = 13$$

$$\begin{aligned} 2x + 3y &= 6 \\ 4x - 5y &= 20 \end{aligned}$$

$$\begin{aligned} \textcircled{1} \quad 2x + 3y &= 6 \\ 2x &= \frac{-3y + 6}{2} \\ x &= -\frac{3}{2}y + 3 \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad 4\left(-\frac{3}{2}y + 3\right) - 5y &= 20 \\ -6y + 12 - 5y &= 20 \\ -11y + 12 &= 20 \\ -11y &= 8 \rightarrow y = -\frac{8}{11} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad 2x + 3y &= 6 \\ 2x + 3\left(-\frac{8}{11}\right) &= 6 \\ \text{''} \quad \left(2x - \frac{24}{11} = 6\right) \end{aligned}$$

$$\begin{aligned} 22x - 24 &= 66 \\ +24 \quad +24 & \end{aligned}$$

$$22x = 90$$

$$x = \frac{90}{22}$$

$$x = \frac{45}{11}$$

$$\left\{ \left(\frac{45}{11}, -\frac{8}{11} \right) \right\}$$