

Graphs of Quadratic Equations

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Focus: Graph an equation of the form

$$y = a(x - h)^2 + k$$

Omit: Domain & Range

Quadratic Equations: $ax^2 + bx + c = 0$

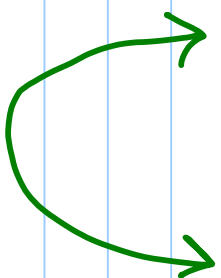
Quadratic functions: $f(x)$

$$y = ax^2 + bx + c$$

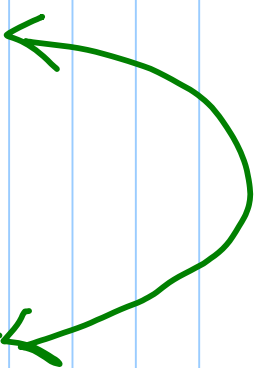
$$y = a(x-h)^2 + k$$

Graphs of quadratic functions:

parabolas

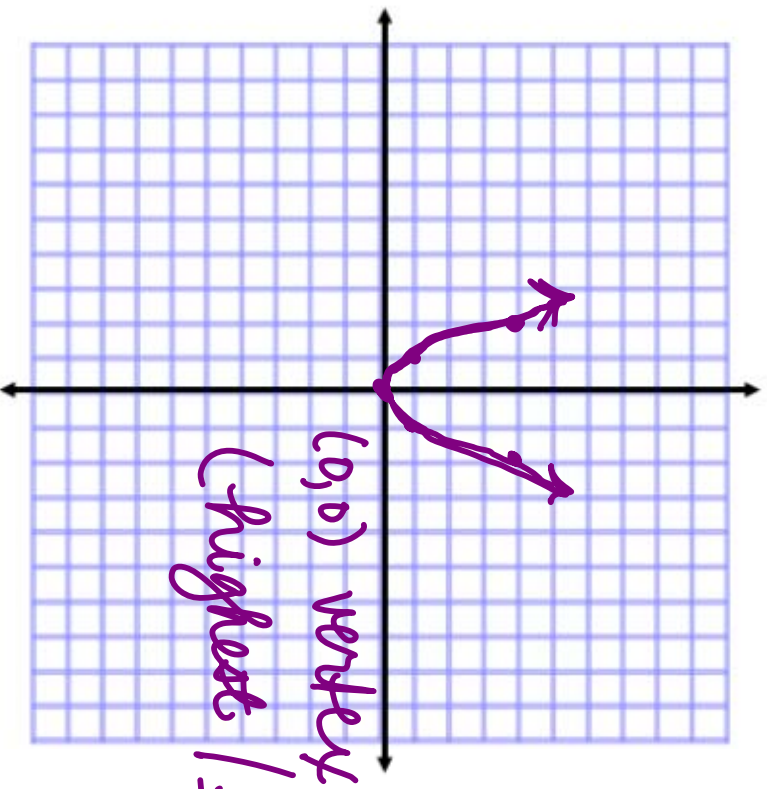


opens up



opens down

Basic Quadratic Function! $y = x^2$

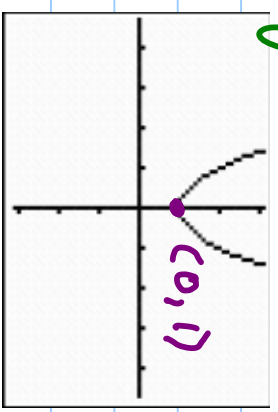


| X | y |
|----|--------------|
| -2 | $(-2)^2 = 4$ |
| -1 | $(-1)^2 = 1$ |
| 0 | $(0)^2 = 0$ |
| 1 | $(1)^2 = 1$ |
| 2 | $(2)^2 = 4$ |

(0,0) vertex
(highest / lowest point)

Graphs of the form: $y = x^2 + k$, k a real #.

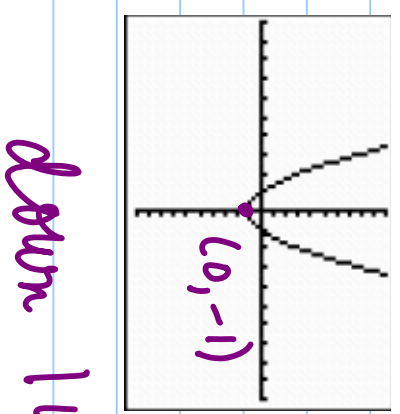
① $y = x^2 + 1$
up 1



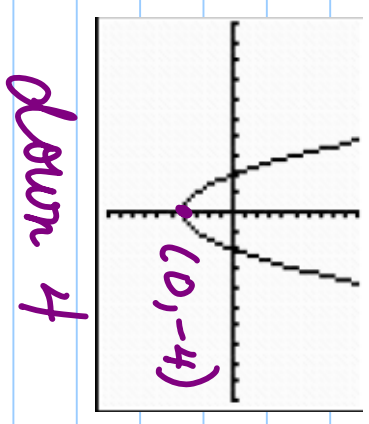
② $y = x^2 + 4$



③ $y = x^2 - 1$



④ $y = x^2 - 4$



Conclusion:

$y = x^2 + k$ has vertex $(0, k)$.

If $k > 0$, shifts up k units.

If $k < 0$, shifts down k units.

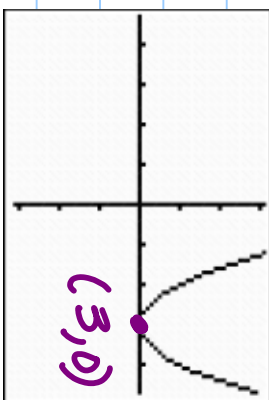
Graphs of the form: $y = (x - h)^2$, h a real #.

Ex.

$$y = (x - 3)^2$$

$$x - 3 = 0$$
$$x = 3$$

$$h = 3$$

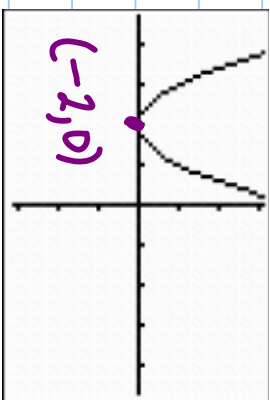


shift
right 3

$$y = (x + 2)^2$$

$$x + 2 = 0$$
$$x = -2$$

$$h = -2$$



shift
left 2

Conclusion: $y = (x-h)^2$ has vertex $(h, 0)$.

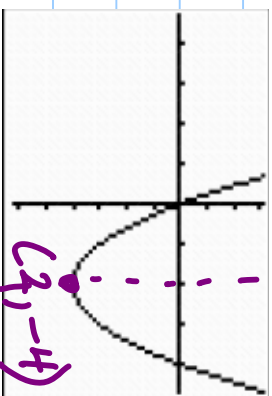
If $h > 0$, shift right. $(x - \#)^2$

If $h < 0$, shift left $(x - (-\#))^2 = (x + \#)^2$

$y = x^2 - 5 \Rightarrow (0, -5)$ } vertex
 $y = (x-5)^2 \Rightarrow (5, 0)$

Graphs of form: $y = (x - \underline{h})^2 + \underline{k}$

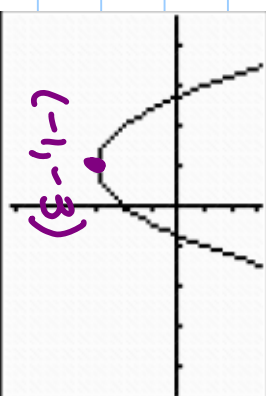
Ex $y = (x - 2)^2 - 4$
 $h = 2, k = -4$



vertex
 $(\underline{h}, \underline{k})$

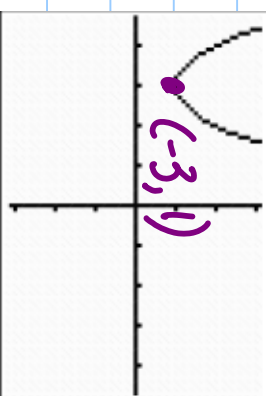
$$y = (x + 1)^2 - 3$$

$h = -1, k = -3$



$$y = (x + 3)^2 + 1$$

$h = -3, k = 1$



Conclusion:

$y = (x-h)^2 + k$ has vertex

(h, k) .

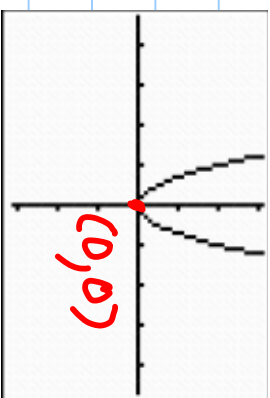
$$y = (x+2)^2 - 6 \quad (-2, -6) \quad y = (x-9)^2 \quad (9, 0)$$

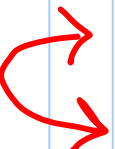
$$y = (x-6)^2 - 2 \quad (6, -2) \quad y = x^2 - 7 \quad (0, -7)$$

$$y = (x+4)^2 + 3 \quad (-4, 3)$$

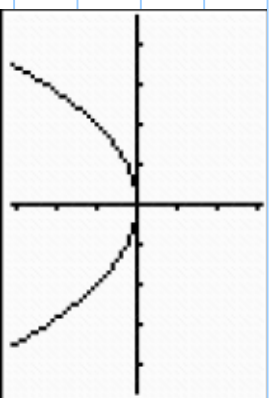
Graphs of form $y = ax^2$, a is a real #.


$$y = \underline{a}x^2$$



a positive
 \Rightarrow 

$$y = \underline{-a}x^2$$



a negative
 \Rightarrow 

$$y = a \overset{\text{direction}}{\downarrow} (x - h) \overset{\text{peak height}}{a} + \underset{\text{y-intercept}}{\textcircled{k}}$$

$$a > 0 \Rightarrow \curvearrowright$$

$$a < 0 \Rightarrow \curvearrowleft$$

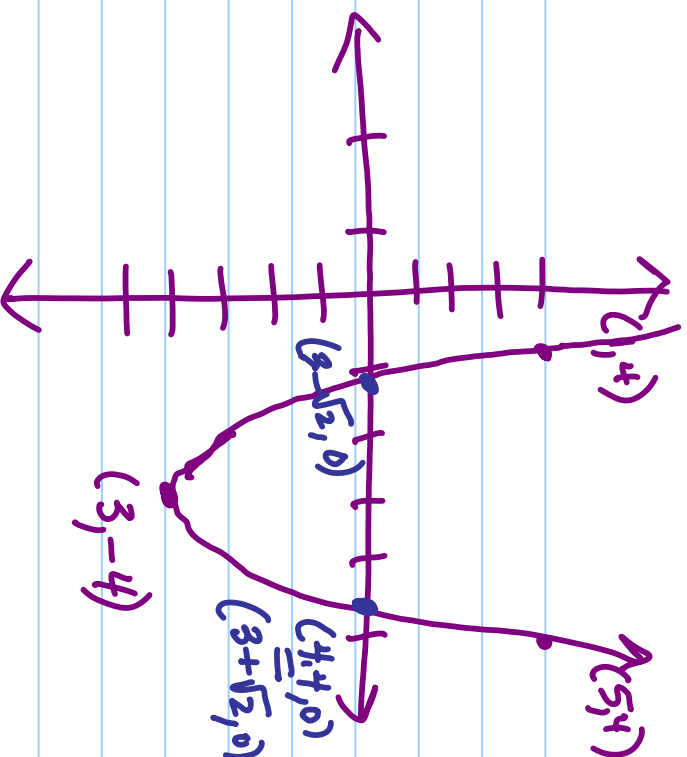
$$\text{vertex} = (h, k)$$

Graph by hand:

$$y = 2(x - 3)^2 - 4$$

$$a = 2 > 0 \quad \curvearrowright$$

$$h = 3, k = -4 \Rightarrow \text{Vertex } (3, -4)$$



$$\begin{array}{r|l} x & y \\ \hline 1 & 4 \\ 3 & -4 \\ 5 & 4 \end{array}$$

y-int:

$$y = 2(0 - 3)^2 - 4$$

$$y = 2(-3)^2 - 4$$

$$y = 2(9) - 4 = 14$$

$$(0, 14)$$

x-int:

$$2(x - 3)^2 - 4 = 0$$

$$2(x - 3)^2 = 4$$

$$(x - 3)^2 = 2$$

$$x - 3 = \pm \sqrt{2}$$

$$x = 3 \pm \sqrt{2}$$

$$y = 2(1 - 3)^2 - 4 = 2(-2)^2 - 4 = 2(4) - 4 = 4$$

$$y = 2(5 - 3)^2 - 4 = 2(2)^2 - 4 = 2(4) - 4 = 4$$

$$y = 2(x - 3)^2 - 4$$

