



Solve Equations Quadratic in Form

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Equations Quadratic in Form

- Equations quadratic in form are equations that may be written as a quadratic equation.
- Our focus will be on rational, radical, and factorable higher degree polynomial equations.

Rational Equations

- Rational equations are equations that contain fractions with variables in the denominator.
- To solve a rational equation, multiply both sides by the LCD and solve the resulting equation.
- Exclude any solutions that make the denominator of the rational equation zero.

Examples – Rational Equation

$$\frac{14}{x} = x - 5 \quad \text{LCD} = x$$
$$x \left(\frac{14}{x} \right) = x(x - 5)$$
$$14 = x^2 - 5x$$
$$0 = x^2 - 5x - 14$$
$$0 = (x - 7)(x + 2)$$
$$x - 7 = 0 \quad \text{or} \quad x + 2 = 0$$
$$x = 7 \quad \quad \quad x = -2$$
$$\{-2, 7\}$$

Example – Rational Equation

$$3 - \frac{1}{t} = \frac{2}{t^2}$$

$$3(t^2) - \frac{1}{t}(t^2) = \frac{2}{t^2}(t^2)$$

$$3t^2 - t = 2$$

$$3t^2 - t - 2 = 0$$

$$a = 3, b = -1, c = -2$$

$$\left\{ -\frac{2}{3}, 1 \right\}$$

$$\text{LCD} = t^2$$

$$t = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-2)}}{2(3)}$$

$$t = \frac{1 \pm \sqrt{1 + 24}}{6}$$

$$t = \frac{1 \pm \sqrt{25}}{6} = \frac{1 \pm 5}{6}$$

$$t = \frac{1+5}{6} = \frac{6}{6} = 1, \quad t = \frac{1-5}{6} = \frac{-4}{6} = -\frac{2}{3}$$

Radical Equations

- Radical Equations are equations containing radicals.
- To solve radical equations, isolate the radical expression to one side and square both sides.
- Solve the resulting equation and check the solutions in the original equation.

Example – Radical Equation

$$2x = \sqrt{11x+3}$$

$$(2x)^2 = (\sqrt{11x+3})^2$$

$$4x^2 = 11x+3$$

$$4x^2 - 11x - 3 = 0$$

$$\frac{12}{+12}$$

$$\{3\}$$

$$4x^2 - 12x + x - 3 = 0$$

$$4x(x-3) + 1(x-3) = 0$$

$$(x-3)(4x+1) = 0$$

$$x-3=0 \text{ or } 4x+1=0$$

$$x=3 \quad 4x=-1$$

$$x=-\frac{1}{4}$$

$$\text{ck: } 2(3) = \sqrt{11(3)+3}$$

$$6 = \sqrt{36}$$

$$6 = 6$$

$$2\left(-\frac{1}{4}\right) = \sqrt{11\left(-\frac{1}{4}\right)+3}$$

$$-\frac{1}{2} = \sqrt{-\frac{11}{4}+3}$$

$$-\frac{1}{2} = \sqrt{\frac{11}{4}}$$

Another Example – Radical Equation

$$p - 2\sqrt{p} = 8$$

$$(p-8)(p-8)$$

$$p-8 = 2\sqrt{p}$$

$$(p-8)^2 = (2\sqrt{p})^2$$

$$p^2 - 16p + 64 = 4p$$

$$-4p \quad -4p$$

$$p^2 - 20p + 64 = 0$$

$$p^2 - 20p + 100 = -64 + 100$$

$$\{16\}$$

$$(p-10)^2 = 36$$

$$p-10 = \pm\sqrt{36}$$

$$p-10 = \pm 6$$

$$p = 10 \pm 6$$

$$p = 10+6, p = 10-6$$

$$p = 16, p = 4$$

$$16 - 2\sqrt{16} \stackrel{?}{=} 8$$

$$16 - 8 \stackrel{?}{=} 8$$

$$8 = 8$$

$$4 - 2\sqrt{4} \stackrel{?}{=} 8$$

$$4 - 4 \stackrel{?}{=} 8$$

$$0 \neq 8$$

Another example – Radical Equation

$$r = \sqrt{\frac{20-19r}{6}}$$

$$(r)^2 = \left(\sqrt{\frac{20-19r}{6}}\right)^2$$

$$r^2 = \frac{20-19r}{6}$$

$$6r^2 = 20-19r$$

$$6r^2 + 19r - 20 = 0$$

ck: $-4 = \frac{20-19(-4)}{6}$

$$6 \cdot 20 = 120 \quad \left\{ \frac{5}{6} \right\}$$

$$+ (24 \cdot 5)$$

$$6r^2 + 24r - 5r - 20 = 0$$

$$6r(r+4) - 5(r+4) = 0$$

$$(r+4)(6r-5) = 0$$

$$r+4=0 \quad \text{or} \quad 6r-5=0$$

$$r = -4 \quad \text{or} \quad 6r = 5$$

$$\frac{5}{6} = \sqrt{\frac{20-19 \cdot \frac{5}{6}}{6}} = \sqrt{\frac{25/6}{6}} = \sqrt{\frac{25}{36}}$$

Higher Degree Polynomial Equations

$$2x^2 + 3x - 5 = 0$$

$$x - 6 = 7$$

- Higher degree polynomial equations are polynomial equations with a power greater than 2.
- To solve higher degree polynomial equations, set the equation equal to zero, factor, and set each factor = 0.
- The degree of the equation tells you how many solutions the equation will have (this includes repetitions).

Example – Polynomial Equation

$$x^4 - 13x^2 + 36 = 0$$

$$(x^2 - 9)(x^2 - 4) = 0$$

$$x^2 - 9 = 0 \quad \text{or} \quad x^2 - 4 = 0$$

$$x^2 = 9$$

$$x^2 = 4$$

$$x = \pm 3$$

$$x = \pm 2$$

$$\{-3, -2, 2, 3\}$$

Another example – Polynomial Equation

$$4k^4 + 5k^2 = -1$$

$$4k^4 + 5k^2 + 1 = 0$$

$$\frac{4 \cdot 1 = 4}{1, 4}$$

$$4k^4 + k^2 + 4k^2 + 1 = 0$$

$$k^2(4k^2 + 1) + 1(4k^2 + 1) = 0$$

$$(4k^2 + 1)(k^2 + 1) = 0$$

Sq Root Prop

$$4k^2 + 1 = 0 \quad \text{or} \quad k^2 + 1 = 0$$

$$4k^2 = -1$$

$$k^2 = -1$$

$$k^2 = -\frac{1}{4}$$

$$k = \pm \sqrt{-1}$$

$$k = \pm \sqrt{-\frac{1}{4}}$$

$$k = \pm i$$

$$k = \pm i \cdot \frac{1}{2}$$

$$k = \pm \frac{i}{2}$$

$$\left\{ \frac{i}{2}, -\frac{i}{2}, i, -i \right\}$$