Solve Quadratic Equations Using the Quadratic Formula

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Review of the Methods to Solve Quadratic Equations

- Solve by factoring – factoring does not solve all equations. (rational #'s)
- Solve by Square Root Property – only works on equations of the form (   )^2 = a number
- Solve by completing the square – solves all equations but can be "messy"

Another Method – The Quadratic Formula

- The quadratic formula is a formula that is derived from completing the square.
- The formula is a method to solve a quadratic equation based on the values of a, b, and c, in the equation

  ax^2 + bx + c = 0.
- This method solves all quadratic equations after written in standard form.

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

$$\text{where } ax^2 + bx + c = 0$$

Examples – Solve using the quadratic formula

$(m-3)(m-5)=0$

$m=3,\ m=5$

$$m^2-8m+15=0$$

$a=1,\ b=-8,\ c=15$

$m=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$m=\dfrac{-(-8)\pm\sqrt{(-8)^2-4(1)(15)}}{2(1)}$

$\{3,5\}$

$m=\dfrac{8\pm\sqrt{64-60}}{2}$

$m=\dfrac{8\pm\sqrt{4}}{2}$

$m=\dfrac{8\pm 2}{2}$

$m=4\pm 1$

$m=4+1=5,\ m=4-1$
$m=3$

Another example

$$26r-2=3r^2$$

$-3r^2+26r-2=0$

$3r^2-26r+2=0$

$a=3,\ b=-26,\ c=2$

$r=\dfrac{-(-26)\pm\sqrt{(-26)^2-4(3)(2)}}{2(3)}$

$r=\dfrac{26\pm\sqrt{676-24}}{6}$

$r=\dfrac{26\pm\sqrt{652}}{6}$

$r=\dfrac{26\pm\sqrt{4\cdot 163}}{6}$

$r=\dfrac{26\pm 2\sqrt{163}}{6}$

$r=\dfrac{13\pm\sqrt{163}}{3}$

$r=\dfrac{13+\sqrt{163}}{3},\ \dfrac{13-\sqrt{163}}{3}$

$r\approx 8.59 \quad r\approx .08$

Another example

$$(k+1)(k-7)=1$$

$k^2-7k+k-7=1$

$k^2-6k-8=0$

$a=1,\ b=-6,\ c=-8$

$k=\dfrac{-(-6)\pm\sqrt{(-6)^2-4(1)(-8)}}{2(1)}$

$k=\dfrac{6\pm\sqrt{36+32}}{2}$

$k=\dfrac{6\pm\sqrt{68}}{2}$

$k=\dfrac{6\pm\sqrt{4\cdot17}}{2}$

$k=\dfrac{6\pm2\sqrt{17}}{2}$

$k=3\pm\sqrt{17}$

$\{3+\sqrt{17},\ 3-\sqrt{17}\}$

Another example

$$r^2-6r+14=0$$

$a=1,\ b=-6,\ c=14$

$r=\dfrac{-(-6)\pm\sqrt{(-6)^2-4(1)(14)}}{2(1)}$

$r=\dfrac{6\pm\sqrt{36-56}}{2}$

$r=\dfrac{6\pm\sqrt{-20}}{2}$

$r=\dfrac{6\pm\sqrt{-4\cdot5}}{2}$

$r=\dfrac{6\pm2i\sqrt{5}}{2}$

$r=3\pm i\sqrt{5}$

$\{3+i\sqrt{5},\ 3-i\sqrt{5}\}$

imaginary

Another example

$$p = \frac{5(5-p)}{3(p+1)}$$

$$\frac{p}{1} = \frac{25-5p}{3p+3}$$

$$3p^2+3p = 25-5p$$
$$+5p \qquad +5p$$

$$3p^2+8p-25=0$$

$a=3,\ b=8,\ c=-25$

$$p = \frac{-(8) \pm \sqrt{(8)^2-4(3)(-25)}}{2(3)}$$

$$p = \frac{-8 \pm \sqrt{64+300}}{6}$$

$$p = \frac{-8 \pm \sqrt{364}}{6}$$

$$p = \frac{-8 \pm \sqrt{4 \cdot 91}}{6}$$

$$p = \frac{-8 \pm 2\sqrt{91}}{6} = \frac{-4 \pm \sqrt{91}}{3}$$

$$\left\{ \frac{-4+\sqrt{91}}{3}, \frac{-4-\sqrt{91}}{3} \right\}$$

The Discriminant

- The discriminant of a quadratic equation is a number that determines the types of solutions. It is the number inside the radical of the quadratic formula, that is the number

$$b^2 - 4ac$$

Implications of discriminant

- If the discriminant is positive, there are two real solutions.
  - If the discriminant is a positive, perfect square, there are two rational solutions. If it is positive and not a perfect square, there are two irrational solutions.

  $\Rightarrow$ factorable
- If the discriminant is zero, there is one real solution (which is rational). $\frac{0}{0}$
- If the discriminant is negative, there are two imaginary solutions.

Find the discriminant and use it to determine the types of solutions.

$$4k^2 - 28k + 49 = 0$$

$a = 4,\ b = -28,\ c = 49$

$b^2 - 4ac$

$(-28)^2 - 4(4)(49)$

$784 - 784$

$0 \Rightarrow$ one rational sol.

$$4x^2 = 4x + 3$$

$4x^2 - 4x - 3 = 0$

$a = 4,\ b = -4,\ c = -3$

$b^2 - 4ac$

$(-4)^2 - 4(4)(-3)$

$16 + 48$

$64 \Rightarrow$ two rational sol.