

## Solving Quadratic Equations by Completing the Square

Andrea Hendricks  
Georgia Perimeter College

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### Example

$$x^2 - 8x + 16 = 3$$

*perfect sq trinomial = #*

$$x^2 - 8x + 16 = 0$$
$$(x-4)(x-4) = 3$$
$$(x-4)^2 = 3$$
$$x-4 = \pm\sqrt{3}$$
$$x = 4 \pm \sqrt{3}$$

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### Perfect Square Trinomials

*⇒ binomial squared*

$$(x+5)^2 = (x+5)(x+5) = x^2 + 10x + 25$$
$$(x-7)^2 = (x-7)(x-7) = x^2 - 14x + 49$$
$$(x-3)^2 = (x-3)(x-3) = x^2 - 6x + 9$$

What is the relationship of the constant term to the middle coefficient?

$$\left(\frac{\text{half of the middle coeff.}}{2}\right)^2$$

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## Complete the Square

Given a binomial of the form  $x^2 + bx$ , a perfect square trinomial may be obtained

by adding  $\left(\frac{b}{2}\right)^2$ . This process is called

completing the square.

$$x^2 + 4x + 4 = (x+2)(x+2)$$

$$x^2 - 12x + 36 = (x-6)(x-6)$$

$$x^2 + 18x + 81 = (x+9)(x+9)$$

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## Completing the Square to solve quadratic equations

- 1) Get terms with the variable on one side of the equation and the constants on the other.
- 2) The leading coefficient on the variable must equal 1. If not, divide by the coefficient.
- 3) Complete the square on the left and add this number to both sides.
- 4) Factor the perfect square trinomial as a binomial squared and simplify the other side.
- 5) Apply the Square Root Property to solve the equation.

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## Examples – Solve by Completing the Square

$$x^2 - 2x + 5 = 9$$

$$\begin{array}{r} x^2 - 2x + 1 = 4 + 1 \\ (x-1)(x-1) = 5 \\ (x-1)^2 = 5 \\ x-1 = \pm\sqrt{5} \\ x = 1 \pm\sqrt{5} \end{array}$$

$$\{1+\sqrt{5}, 1-\sqrt{5}\}$$

$$\{3.24, -1.24\}$$

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Examples – Solve by  
Completing the Square

$$y^2 + 10y + 18 = 0$$

$$y^2 + 10y + 25 = -18 + 25$$

$$(y+5)^2 = 7$$

$$y+5 = \pm\sqrt{7}$$

$$y = -5 \pm \sqrt{7}$$

$$\{-5 + \sqrt{7}, -5 - \sqrt{7}\}$$

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Example

$$3x^2 - 24x - 21 = 0$$

$$\frac{3x^2 - 24x}{3} = \frac{21}{3}$$

$$x^2 - 8x + 16 = 7 + 16$$

$$(x-4)^2 = 23$$

$$x-4 = \pm\sqrt{23}$$

$$x = 4 \pm \sqrt{23}$$

$$\{4 + \sqrt{23}, 4 - \sqrt{23}\}$$

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