

# Complex Numbers

Andrea Hendricks  
Georgia Perimeter College

Review: Simplify:

$$\sqrt{16} = 4$$

$$\sqrt{4} = 2$$

$$\sqrt{81} = 9$$

$$\sqrt{-36} \text{ no real \#}$$

$$4^2 = 16$$

$$2^2 = 4$$

$$9^2 = 81$$

$$(\quad)^2 = -36$$

$$(-6)^2 = 36$$

Definitions:  $\sqrt{-1} = i, i^2 = -1$   
 $\sqrt{9} = 3, 3^2 = 9$

Simplify:

$$\sqrt{-36} = \sqrt{36 \cdot -1} = 6i$$

$$\sqrt{-81} = \sqrt{81 \cdot -1} = 9i$$

$$5 + \sqrt{-9} = 5 + \sqrt{9 \cdot -1} = 5 + 3i$$

$$\sqrt{-7} = i\sqrt{7}$$
$$\sqrt{7}i$$

Definition :  $\sqrt{-b} = i\sqrt{b}$

Examples : Perform the operations

$$\sqrt{-4} \cdot \sqrt{-9} = (2i)(3i) = 6i^2 = 6(-1) = -6$$

$$\sqrt{-3} \cdot \sqrt{5} = i\sqrt{3 \cdot 5} = i\sqrt{15}$$

$$\frac{\sqrt{-20}}{\sqrt{5}} = \frac{\sqrt{-4.5}}{\sqrt{5}} = \frac{2i\sqrt{5}}{\sqrt{5}} = 2i$$

Def. Any number written in the form  $a+bi$ , where  $a, b$  are real #'s and  $i = \sqrt{-1}$ , is a complex #.

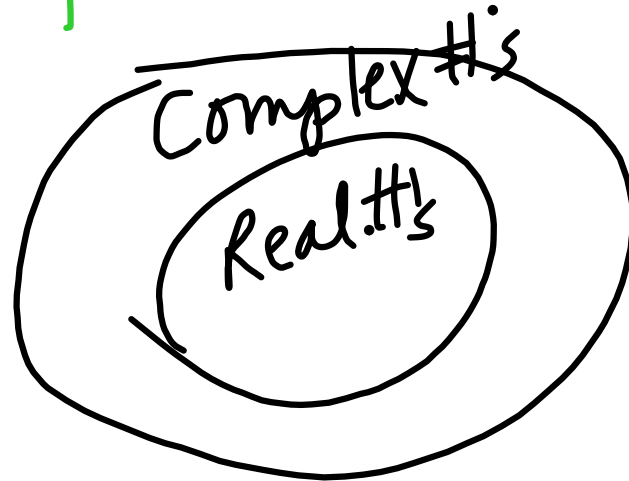
standard form

$5+2i$   
 $3-7i$

Note  $5 = 5 + 0i \Rightarrow$  All real numbers are complex #'s.

5 = real part

0 = imag. part



Any operation performed on real #'s may be performed with complex #'s.

# Add / Subtract Complex #'s: combining like terms

$$(3+4i) + (7+6i)$$

$$\underline{3} + \underline{\underline{4i}} + \underline{7} + \underline{\underline{6i}} = 10 + 10i$$

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$$(3-4i) + (7-6i)$$

$$\underline{3} - \underline{\underline{4i}} + \underline{7} - \underline{\underline{6i}} = 10 - 10i$$

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$$(3-4i) - (7-6i)$$

$$\underline{3} - \underline{\underline{4i}} - \underline{7} + \underline{\underline{6i}} = -4 + 2i$$

# Multiply Complex #'s $(a+bi)$

$$\sqrt{-1} = i$$
$$i^2 = -1$$

$$5i(2+3i) = 10i + 15i^2$$
$$= 10i + 15(-1)$$
$$= 10i - 15 \text{ or } -15 + 10i$$

$$(4-2i)(1-7i)$$

$$4 - 28i - 2i + 14i^2$$

$$4 - 30i + 14(-1) = 4 - 30i - 14 = -10 - 30i$$

$$(5-3i)(5+3i)$$

$$25 + 15i - 15i - 9i^2$$

$$25 - 9(-1) = 25 + 9 = \underline{\underline{34}}$$

product of  
conjugates

$a+bi$   
 $a-bi$

$$\begin{aligned}(5-3i)^2 &= (5-3i)(5-3i) \\ &= 25 - 15i - 15i + 9i^2 \\ &= 25 - 30i - 9 \\ &= 16 - 30i\end{aligned}$$

\* Omit dividing complex #'s :  
powers of  $i$ .

# Calculator Usage

2<sup>nd</sup>  $\boxed{\cdot}$   $\Rightarrow i$

$\boxed{\text{Mode}}$   $\downarrow$  Real  $\rightarrow a+bi$   $\boxed{\text{Enter}}$

$$\sqrt{-36} = 6i$$