Simplifying Radical Expressions

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Objectives

- Use the product rule for radicals
- Use the quotient rule for radicals
- Simplify radicals
- Use the Pythagorean Theorem
- Use the distance formula

Objective 1: Use the product rule for radicals

$\sqrt{4 \cdot 9}$

$\sqrt{36}$

6

$\sqrt{4} \cdot \sqrt{9}$

$2 \cdot 3$

6

$\sqrt{4 \cdot 9} = \sqrt{4} \cdot \sqrt{9}$

Product Rule for Radicals

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$$

same index $\Rightarrow$ multiply radicals

Note: $\sqrt[n]{ab} = (ab)^{1/n} = a^{1/n} b^{1/n} = \sqrt[n]{a}\sqrt[n]{b}$

Examples: Use the product rule

$\sqrt{5} \cdot \sqrt{6} = \sqrt{30}$

$\sqrt{9} \cdot \sqrt{2} = \sqrt{18}$

$3\sqrt{2}$

$\sqrt{14} \cdot \sqrt{x} = \sqrt{14x}$

$\sqrt[3]{7x} \cdot \sqrt[3]{2y} = \sqrt[3]{14xy}$

Objective 2: Use the quotient rule for radicals

$$\sqrt[3]{\frac{8}{27}} = \frac{2}{3}$$

$$\frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{2}{3}$$

$\left(\frac{2}{3}\right)^3 = \frac{8}{27}$

$\sqrt[3]{\frac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}}$

Quotient Rule for Radicals

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$\left(\frac{a}{b}\right)^{1/n} = \frac{a^{1/n}}{b^{1/n}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Examples: Use the quotient rule.

$$\sqrt{\frac{64}{121}} = \frac{\sqrt{64}}{\sqrt{121}} = \frac{8}{11}$$

$$\sqrt{\frac{x}{25}} = \frac{\sqrt{x}}{\sqrt{25}} = \frac{\sqrt{x}}{5}$$

$$\sqrt{\frac{13}{49}} = \frac{\sqrt{13}}{\sqrt{49}} = \frac{\sqrt{13}}{7}$$

$$\sqrt{\frac{p^6}{81}} = \frac{\sqrt{p^6}}{\sqrt{81}} = \frac{p^3}{9}$$

$$(x^2)^3 = x^6$$

$$\sqrt[3]{\frac{x^6}{8}} = \frac{\sqrt[3]{x^6}}{\sqrt[3]{8}} = \frac{x^2}{2}$$

$$\sqrt[4]{\frac{81}{x^4}} = \frac{\sqrt[4]{81}}{\sqrt[4]{x^4}} = \frac{3}{x}$$

Objective 3: Simplify Radicals

For a radical to be simplified, it must satisfy these conditions.

1) The radicand has no factor raised to a power greater than or equal to the index.
2) The radicand has no fractions.
3) No denominator contains a radical.
4) Exponents in the radicand and the index of the radical have no common factor other than 1.

Simplify the radicals

54: 1,54 | 2,27 | 3,18 | 6,9

12: 1,12 | 2,6 | $\boxed{3,4}$

$\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$

72: 1,72 | 2,36 | 3,24 | 4,18 | 6,12 | 8,9

$\sqrt{72} = \sqrt{36 \cdot 2} = \sqrt{36} \cdot \sqrt{2} = 6\sqrt{2}$

48: 1,48 | 2,24 | 3,16 | 4,12 | 6,8

$\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$

$\sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = \sqrt[3]{27} \cdot \sqrt[3]{2} = 3\sqrt[3]{2}$

More examples

$$\sqrt[2]{x^5} = \sqrt{x^4 \cdot x} = \sqrt{x^4} \cdot \sqrt{x}$$
$$= x^2\sqrt{x}$$

x^5

$x \cdot x^4$

$x^2 \cdot x^3$

$$\sqrt{18m^2} = \sqrt{9 \cdot 2m^2}$$
$$= \sqrt{9m^2} \cdot \sqrt{2}$$
$$= 3m\sqrt{2}$$

18

1, 18

2, 9

3, 6

More examples

$$\sqrt[3]{-8x^8} = \sqrt[3]{-8x^6 \cdot x^2}$$
$$= \sqrt[3]{-8x^6} \cdot \sqrt[3]{x^2}$$
$$= -2x^2\sqrt[3]{x^2}$$

x^8

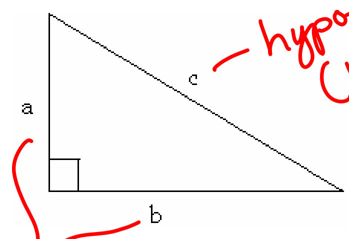
$x \cdot x^7$

$x^2 \cdot x^6$

$x^3 \cdot x^5$

$x^4 \cdot x^4$

Objective 4: Use the Pythagorean Theorem

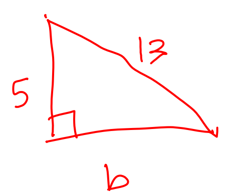


hypotenuse (longest)

legs

$$a^2 + b^2 = c^2$$

Example: Use the Pythagorean Theorem to find the missing value.

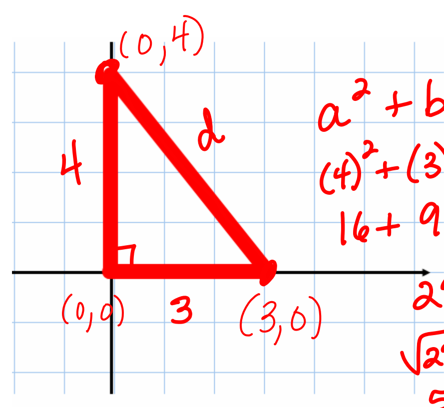


$$a^2 + b^2 = c^2$$
$$(5)^2 + b^2 = (13)^2$$
$$25 + b^2 = 169$$
$$b^2 = 144$$
$$b = \sqrt{144}$$
$$b = 12$$

Objective 5: Use the distance formula.

The distance formula is used the find the distance or length of a line segment between two points on the coordinate system.

Find the distance between (3,0) and (0,4).



$a^2 + b^2 = c^2$

$(4)^2 + (3)^2 = d^2$

$16 + 9 = d^2$

$25 = d^2$

$\sqrt{25} = d$

$5 = d$

Distance Formula

The distance between (x1, y1) and (x2, y2) is given by

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

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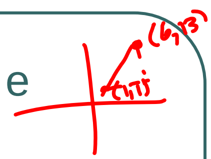
Examples: Find the distance between the two points.



① $(6,13)$ and $(1,1)$ — $x_1\ y_1$, $x_2\ y_2$

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$d = \sqrt{(6-1)^2 + (13-1)^2}$

$d = \sqrt{(5)^2 + (12)^2}$

$d = \sqrt{25 + 144}$

$d = \sqrt{169} = 13$

② $(-6,5)$ and $(3,-4)$ — $x_1\ y_1$, $x_2\ y_2$

$d = \sqrt{(-6-3)^2 + (5-(-4))^2}$

$d = \sqrt{(-9)^2 + (9)^2} = \sqrt{81+81}$

$d = \sqrt{162}$

$d = \sqrt{81 \cdot 2}$

$d = \sqrt{81} \cdot \sqrt{2}$

$d = 9\sqrt{2}$