

Rational Exponents

Andrea Hendricks
Georgia Perimeter College

Objectives

- Use exponential notation for nth roots
- Define and use expressions with exponents of the form $a^{m/n}$
- Convert between radicals and rational exponents.
- Use the rules for exponents with rational exponents

Objective 1: Use exponential notation for nth roots

Let's use the calculator to investigate some exponential expressions.

$$4^{(1/2)}$$

2

$$27^{(1/3)}$$

3

$$81^{(1/4)}$$

3

$$\begin{array}{l} \sqrt{4} = 2 \quad 4^{1/2} = \sqrt{4} \\ \sqrt[3]{27} = 3 \quad 27^{1/3} = \sqrt[3]{27} \\ \sqrt[4]{81} = 3 \quad 81^{1/4} = \sqrt[4]{81} \end{array}$$

How are powers of the form $1/n$ related to radicals?

$$a^{1/n} = \sqrt[n]{a}$$

Examples: Evaluate the expressions.

$$16^{1/2} = \sqrt{16} = 4$$

$$-16^{1/2} = -\sqrt{16} = -4$$

$$(-16)^{1/2} = \sqrt{-16}$$

not a real #

$$\left(\frac{64}{125}\right)^{1/3} = \sqrt[3]{\frac{64}{125}} = \frac{4}{5}$$

$$(-8)^{1/3} = \sqrt[3]{-8} = -2$$

Objective 2: Use powers of the form m/n

m power n index

$$a^{1/n} = \sqrt[n]{a}$$

$4^{(3/2)}$	8	$(\sqrt[3]{27})^4$	81
$27^{(4/3)}$	81		
$(\sqrt{4})^3$	8		

$$4^{(3/2)} = 8 \quad 27^{4/3} = 81 \quad (\sqrt{4})^3 = 2^3 = 8 \quad (\sqrt[3]{27})^4 = 3^4 = 81$$

Definition of $a^{m/n}$

$$a^{m/n} = \left(\sqrt[n]{a} \right)^m = \sqrt[n]{a^m}$$

Examples: Evaluate the exponential expressions.

$$100^{(3 \div 2)}$$

$$100^{3/2} = \left(\sqrt{100} \right)^3 = (10)^3 = 1000$$
$$-32^{3/5} = - \left(\sqrt[5]{32} \right)^3 = -(2)^3 = -8$$
$$64^{-2/3} = \left(\sqrt[3]{64} \right)^{-2} = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

Objective 3: Convert between radicals and rational exponents.

Radical Form

$$\sqrt{x}$$

$$\sqrt[3]{3x}$$

$$4\sqrt[3]{x^2}$$

Exponential Form

$$x^{1/2}$$

$$(3x)^{1/3}$$

$$4x^{2/3}$$

Objective 4: Use the rules for exponents with rational exponents

$$a^m a^n = a^{m+n}$$

$$(a^m)^n = a^{mn}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(ab)^m = a^m b^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^0 = 1$$

Examples: Use the rules of exponents to simplify.

$$3^{1/2} \cdot 3^{3/2} = 3^{\frac{1}{2} + \frac{3}{2}} = 3^{\frac{4}{2}} = 3^2 = 9$$

$$\frac{64^{5/3}}{64^{4/3}} = 64^{5/3 - 4/3} = 64^{1/3} = \sqrt[3]{64} = 4$$

$$(8x^6)^{1/3} = 8^{1/3} (x^6)^{1/3} = 2x^2$$

$2 \cdot \frac{1}{3}$

Examples: Use the rules of exponents to simplify.

$$\frac{(r^{1/5} s^{2/3})^{15}}{r^2} = \frac{(r^{1/5})^{15} (s^{2/3})^{15}}{r^2}$$

$$= \frac{r^3 s^{10}}{r^2}$$

$$= r s^{10}$$

$\frac{1}{5} \cdot 15 = 3$
 $\frac{2}{3} \cdot 15 = 10$