

Least Common Denominators

The LCD is the simplest expression that is divisible by all denominators.

Recall Find the LCD of $\frac{1}{24}$, $\frac{7}{15}$.

$$24 = 4 \cdot 6 = 2 \cdot 2 \cdot 2 \cdot 3$$

$$15 = 3 \cdot 5$$

$$\text{LCD} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$$

$$= 120$$

Steps to find the LCD:

- ① Factor each denominator completely.
- ② List each different denominator factor the greatest number of times it appears in any of the denominators.
- ③ Multiply the factors in step 2.

Ex Find the LCD for the fractions in each list.

① $\frac{9}{x^2}, \frac{8}{x^5}$

$$x^2 = x \cdot x$$

$$x^5 = x \cdot x \cdot x \cdot x \cdot x$$

$$\left. \begin{array}{l} x^2 = x \cdot x \\ x^5 = x \cdot x \cdot x \cdot x \cdot x \end{array} \right\} \begin{array}{l} \text{LCD} = x \cdot x \cdot x \cdot x \cdot x \\ = x^5 \end{array}$$

$$\textcircled{2} \quad \frac{7}{6p}, \frac{15}{4p-8}$$

$$6p = 2 \cdot 3 \cdot p$$

$$4p-8 = 4(p-2) = \underbrace{2 \cdot 2}_{\text{}}(p-2)$$

$$\text{LCD} = 2 \cdot 2 \cdot 3 \cdot p(p-2)$$

$$\text{LCD} = 12p(p-2)$$

$$\textcircled{3} \quad \frac{7}{5b-10}, \frac{11}{6b-12}$$

$$5b-10 = 5(b-2)$$

$$6b-12 = 6(b-2) = 2 \cdot 3(b-2)$$

$$\begin{aligned} \text{LCD} &= 2 \cdot 3 \cdot 5(b-2) \\ &= 30(b-2) \end{aligned}$$

$$\textcircled{4} \quad \frac{3}{8y+16}, \quad \frac{22}{y^2+3y+2}$$

$$8y+16 = 8(y+2)$$

$$y^2+3y+2 = (y+2)(y+1)$$

$$\text{LCD} = 8(y+2)(y+1)$$

$$\textcircled{5} \quad \frac{7}{z^2 + 2z} \quad , \quad \frac{10}{z^2 - 4}$$

$$z^2 + 2z = z(z + 2)$$

$$z^2 - 4 = (z + 2)(z - 2)$$

$$\text{LCD} = z(z + 2)(z - 2)$$

⑥

$$\frac{12}{m-3}, \frac{-4}{3-m}$$

$$\frac{12}{m-3}, \frac{4}{-(3-m)} = \frac{4}{-3+m} = \frac{4}{m-3}$$

$$\text{LCD} = m - 3$$