

# The Fundamental Property of Rational Expressions

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Def. A rational expression is an expression of the form  $\frac{P}{Q}$ , where  $P$  and  $Q$  are polynomials.

Ex.  $\frac{2x}{x+3}$ ,  $\frac{x^2+2x+1}{x^2-1}$

Objective Evaluate a rational expression.

Ex. Find the value of  $\frac{3}{x+2}$

for

①  $x = 2$       $\frac{3}{2+2} = \frac{3}{4}$

②  $x = 0$       $\frac{3}{0+2} = \frac{3}{2}$

③  $x = -2$       $\frac{3}{-2+2} = \frac{3}{0}$  undefined

Objective Determine when a rational expression is undefined (den.=0).

Ex Find any values of  $x$  for which the expression is undefined.

$$\textcircled{1} \quad \frac{x+1}{x}$$

$$x=0$$

$$\textcircled{2} \quad \frac{x+2}{x-6}$$

$$x-6=0$$
$$x=6$$

$$\textcircled{3} \frac{2r-5}{r^2-5r+4}$$

$$\begin{aligned} r^2 - 5r + 4 &= 0 \\ (r-4)(r-1) &= 0 \\ r-4=0 \quad r-1=0 \\ \textcircled{r=4} \quad \textcircled{r=1} \end{aligned}$$

$$\textcircled{4} \frac{3x-1}{x^2+1}$$

$$x^2 + 1 \neq 0$$

no values

Objective Reduce to lowest terms.

Factor the numerator & denominator.  
Divide out common factors.

$$\frac{P\cancel{K}}{Q\cancel{K}} = \frac{P}{Q}$$

$$\frac{4}{8} = \frac{\cancel{4} \cdot 1}{\cancel{4} \cdot 2} = \frac{1}{2}$$

Ex. Reduce to lowest terms.

$$\textcircled{1} \frac{27P^2}{3P} = \frac{\cancel{3}P(9P)}{\cancel{3}P(1)} = \frac{9P}{1} = 9P$$

$$\textcircled{2} \frac{8z - 24}{4z - 12} = \frac{\cancel{8}^2(z - 3)}{\cancel{4}(z - 3)} = 2$$

$$\textcircled{3} \quad \frac{2t+6}{t^2-9} = \frac{2(\cancel{t+3})}{(\cancel{t+3})(t-3)} = \frac{2}{t-3}$$

$$\textcircled{4} \quad \frac{x^2+2x-15}{x^2+6x+5} = \frac{(\cancel{x+5})(x-3)}{(\cancel{x+5})(x+1)} \\ = \frac{x-3}{x+1}$$

$$\textcircled{5} \quad \frac{7t^2 - 3t - 20}{7t + 4}$$

$$\frac{7t^2 + 4t - 35t - 20}{7t + 4}$$

$$\frac{t(7t+4) - 5(7t+4)}{7t+4}$$

$$\frac{\cancel{7t+4}(t-5)}{\cancel{7t+4}} = t-5$$

$$\left\{ \begin{array}{l} 140 \\ \hline 1,140 \\ 2,70 \\ \hline \textcircled{4,35} \\ \hline 5,28 \\ \vdots \end{array} \right. \begin{array}{l} \text{diff} = 31 \\ \\ +4 - 35 \end{array}$$

Objective

Simplify quotient of opposites.

5 & -5 are opposites.  $\frac{5}{-5} = -1$

What is the opposite of  $x-3$ ?

$$-(x-3) = -x+3 = 3-x$$

Dividing these:  $\frac{x-3}{3-x} = -1$

$$\frac{p-q}{q-p} = -1$$

Simplify:

$$\frac{m^2 - 1}{1 - m}$$

$$= \frac{(m+1)\overset{-1}{\cancel{(m-1)}}}{\cancel{1-m}}$$

$$= -(m+1)$$

$$= -m - 1$$

Objective

Write equivalent forms  
of a rational  
expression.

Recall

$$\begin{array}{l} \Rightarrow \frac{4}{5} = \frac{-4}{5} \text{ or } \frac{4}{-5} \\ \searrow \end{array}$$

$$-\frac{a}{b} = \frac{-a}{b} \text{ or } \frac{a}{-b}$$

Ex. Rewrite

$-\frac{x+4}{x-3}$  in 4 ways.

$$\frac{-(x+4)}{x-3} = \frac{-x-4}{x-3}$$

$$\frac{x+4}{-(x-3)} = \frac{x+4}{3-x}$$