

Solving Quadratic Equations by Factoring

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Definition

A quadratic equation is an equation of the form

$$\underline{a}x^2 + \underline{b}x + \underline{c} = 0, \text{ where } a, b, \text{ and } c \text{ are real numbers}$$

with $a \neq 0$.

standard form

[Zero-Factor property]

If a and b are real numbers and if $ab = 0$,
then $a = 0$ or $b = 0$.

$$3 \cdot \underline{0} = 0$$

$$\underline{0} \cdot 6 = 0$$

[Steps to Solve a Quadratic Equation by Factoring]

- 1) Write the equation in standard form.
(That is, make one side of the equation zero.)
- 2) Factor the polynomial, if necessary.
- 3) Set each factor equal to 0 and solve.

2 solutions (count repetitions)

[Example – Solve the equation.]

$$(x+5)(x-3) = 0$$

$$x+5=0 \text{ or } x-3=0$$

$$x=-5$$

$$x=3$$

$$\{-5, 3\}$$

[Example – Solve the equation.]

$$\underline{2x(x-7)} = 0$$

$$\frac{2x}{2} = 0 \text{ or } x-7=0$$

$$x=7$$

$$x=0$$

$$\{0, 7\}$$

[Example – Solve the equation.]

$$y^2 + 3y - 4 = 0$$

$$(y + 4)(y - 1) = 0$$

$$y + 4 = 0 \quad \text{or} \quad y - 1 = 0$$

$$\begin{matrix} -4 & -4 \\ y = -4 \end{matrix} \quad \begin{matrix} +1 & +1 \\ y = 1 \end{matrix}$$

$$\{-4, 1\}$$

[Example – Solve the equation.]

$$x^2 = 24 - 5x$$

$$x^2 + 5x - 24 = 0$$

$$(x - 3)(x + 8) = 0$$

$$x - 3 = 0 \quad \text{or} \quad x + 8 = 0$$

$$x = 3 \quad \quad \quad x = -8$$

$$\{-8, 3\}$$

$$24$$

$$\text{diff} = 5$$

$$1, 24$$

$$2, 12$$

$$\textcircled{3, 8}$$

$$4, 6$$

$$\text{ck: } 3^2 \stackrel{?}{=} 24 - 5(3)$$

$$9 = 24 - 15 \quad \checkmark$$

[Example – Solve the equation.]

$$12p^2 = 8 - 10p$$

$$12p^2 + 10p - 8 = 0$$

$$12p^2 + 16p - 6p - 8 = 0$$

$$4p(3p + 4) - 2(3p + 4) = 0$$

$$(3p + 4)(4p - 2) = 0$$

$$3p + 4 = 0 \quad \text{or} \quad 4p - 2 = 0$$

$$3p = -4 \quad \text{or} \quad 4p = 2$$

$$p = -\frac{4}{3} \quad \text{or} \quad p = \frac{2}{4} = \frac{1}{2}$$

$$\left\{ -\frac{4}{3}, \frac{1}{2} \right\}$$

$12 \cdot 8 = 96$
 $\frac{96}{12} = 8$
 $1, 96$
 $2, 48$
 $3, 32$
 $4, 24$
 $6, 16$

[Example – Solve the equation.]

$$x^2 = 64x \Rightarrow x^2 - 64x = 0 \quad \{0, 64\}$$

$$x(x - 64) = 0$$

$$x = 0 \quad \text{or} \quad x - 64 = 0$$

$$x = 64$$

$$x^2 = 64$$

$$x^2 - 64 = 0$$

$$(x - 8)(x + 8) = 0$$

$$x - 8 = 0 \quad \text{or} \quad x + 8 = 0$$

$$x = 8 \quad \text{or} \quad x = -8 \quad \{-8, 8\}$$

[Example – Solve the equation.]

$$\begin{aligned}
 r(r-5) &= -6 \\
 r^2 - 5r &= -6 \\
 r^2 - 5r + 6 &= 0 \\
 (r-3)(r-2) &= 0 \\
 r-3=0 \text{ or } r-2=0 \\
 r=3 & \qquad r=2 \\
 & \{2, 3\}
 \end{aligned}$$

[Example – Solve the equation.]

$$\begin{aligned}
 3x(2x+7) &= 12 \\
 6x^2 + 21x &= 12 \\
 6x^2 + 21x - 12 &= 0 \\
 \frac{6x^2 + 21x - 12}{3} & \\
 2x^2 + 7x - 4 &= 0 \\
 2x^2 + 8x - x - 4 &= 0 \\
 2x(x+4) - 1(x+4) &= 0 \\
 (x+4)(2x-1) &= 0 \\
 x+4=0 & \quad 2x-1=0 \\
 x=-4 & \quad 2x=1 \\
 & \quad x=\frac{1}{2} \\
 \{ -4, \frac{1}{2} \} &
 \end{aligned}$$

$2 \cdot 4 = \frac{8}{1,8}$